Leptonic constants of heavy quarkonia in potential approach of NRQCD

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Abstract

We consider a general scheme for calculating the leptonic constant of heavy quarkonium $\bar{Q}Q$ in the framework of nonrelativistic quantum chromodynamics, NRQCD, operating as the effective theory of nonrelativistic heavy quarks. We explore the approach of static potential in QCD, which takes into account both the evolution of effective charge in the three-loop approximation and the linearly raising potential term, which provides the quark confinement. The leptonic constants of $\bar{b}b$ and $\bar{c}c$ systems are evaluated by making use of two-loop anomalous dimension for the current of nonrelativistic quarks, where the factor for the normalization of matrix element is introduced in order to preserve the renormalization group invariance of estimates.

PACS Numbers: 12.39.Pn, 12.39.Jh, 14.40.Gx

1 Introduction

A description of leptonic constants for a heavy quarkonium composed of heavy quark and heavy anti-quark $\bar{Q}Q$ plays an important role in studies of QCD dynamics in the sector of heavy quarks. So, the precision calculations of heavy quark masses $m_{b,c}$ and running coupling constant α_s in QCD are performed in the framework of QCD sum rules [1] and effective theory of nonrelativistic heavy quarks, NRQCD [2]. In such the approach the experimental data on both the masses of S-wave quarkonium states and their leptonic constants [3] are used. The consideration of NRQCD sum rules was done for the bottomonium with taking into account of one-loop gluon corrections in [4] (next-to-leading-order, NLO) as well as with two-loop corrections in [5–7] (next-to-next-to-leading-order, N^{2}LO), and for the charmonium in [8]. In general, the sum rules of NRQCD could be used for the solution of inverse problem, i.e. the calculation of leptonic constants at given

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values of heavy quark masses, a definite normalization of coupling constant and the experimental mass spectrum of quarkonia. The only additional entry is caused by that the analysis in the framework of NRQCD sum rules supposes that the region under study remains correctly defined only if one takes into account the higher excitations of ground state, so that one should fix the ratios of leptonic constants, i.e. their relative weights in the sum rules, along with the mass spectrum. Another way is to go in the region of parameters, where the contributions of ground state excitations are negligible, and the term of gluon condensate becomes significant, while its Wilson coefficient requires cumbersome calculations in higher orders of perturbation theory.

An alternative method is the description of heavy quarkonium in the framework of potential approach. In this way, if the static potential in QCD with infinitely heavy sources in the fundamental representation of color gauge group is given, then in the leading order of NRQCD, i.e., if we neglect relativistic corrections and interactions depending on the quark spins, the heavy quark masses can be determined with a high accuracy from the comparison of experimental mass spectrum of heavy quarkonia $\bar{b}b$ and $\bar{c}c$ with the estimates in the NRQCD. In this procedure, one calculates the spectra of stationary energy levels E for the nonrelativistic Schrödinger equation with the static potential, so that the masses are defined by the expression $M = m_1 + m_2 + E$, where $m_{1,2}$ are the masses of heavy quark and heavy anti-quark composing the quarkonium. In addition, in ref. [9] authors derived the static potential in QCD, so that it takes into account recent perturbative results at short distances [10, 11] as well as the regime of confinement in the infrared region. This potential is straightforward modification of model by Buchmüller and Tye [12], who suggested the method with unified two-loop β-function for the effective charge $\mathfrak{a}$. This function satisfied two definite asymptotic limits at $\mathfrak{a} \to 0$ (the perturbation theory) and $\mathfrak{a} \to \infty$ (the potential confining the quarks in hadrons, in the form of linearly raising term under the distance increase). The calculations in three loops [10, 11] shown that the value and sign of coefficient β_2 modelled in the paper by Buchmüller and Tye are significantly different from the correct results for the β-function. The modification allows us to reach a consistent description for combined data on the mass spectra of heavy quarkonia and the evolution parameter Λ_{QCD} for the running coupling constant measured at high virtuality of m_Z [3]. Thus, in the potential approach we have got a possibility to estimate the leptonic constants of heavy quarkonia in NRQCD with the quark masses and coupling constant α_s adjusted with a high accuracy.

In this way, we consider a general scheme for the calculation of leptonic constants, since the results of NRQCD have to be related with the quantities defined in full QCD. There are two challenges in this respect. The first one is the Wilson coefficient depending on the scale of calculations in NRQCD, which relates the current of nonrelativistic quarks considered in the leading order of inverse heavy quark mass, with the electromagnetic current of heavy quarks in full QCD. An anomalous dimension of this coefficient $\mathcal{K}$ is known with the two-loop accuracy [13–15]. The factor $\mathcal{K}$ is matched with the value, which is given by the equality of currents in QCD and its nonrelativistic approximation at a scale $\mu_{\rm hard}$ about the mass of heavy quark. Therefore, the operator equality of currents in full QCD and NRQCD is completely defined up to the two-loop accuracy. The second challenge is the matrix element for the current of nonrelativistic quarks over the vacuum and physical state of quarkonium. Because of renormalization group invariance of physical quantities, this matrix element has got the anomalous dimension equal to that of the current in NRQCD. For the first time, in the potential approach we perform the consideration involving the factor $\mathcal{A}$ providing the renormalization invariance of results. In this way, we have to introduce a scale of normalization for $\mathcal{A}$, so that the reasonable region of scale change is restricted

by the physical characteristics for the quark system under study. Then, the obtained estimates of leptonic constants for the quarkonia are stable in a region of change for both the QCD-to-NRQCD matching scale μ_{hard} and the point of perturbative calculations in NRQCD μ_{fact}.

In Section 2 we consider the description of leptonic constants for the heavy quarkonia in the framework of NRQCD. The potential approach is actually given in Section 3. The numerical estimates are presented in Section 4. In Conclusion we summarize the obtained results.

2 Leptonic constants

In the NRQCD approximation for the heavy quarks, the calculation of leptonic constant for the heavy quarkonium with the two-loop accuracy requires the matching of NRQCD currents with the currents in full QCD,

$$J_\nu^{QCD} = \bar{Q}\gamma_\nu Q, \quad \mathcal{J}_\nu^{NRQCD} = \chi^\dagger \sigma_\nu^\perp \phi,$$

where we have introduced the following notations: Q is the relativistic quark field, χ and ϕ are the nonrelativistic spinors of anti-quark and quark, $\sigma_\nu^\perp = \sigma_\nu - v_\nu(\sigma\cdot v)$, where v is the four-velocity of heavy quarkonium, so that

$$J_\nu^{QCD} = \mathcal{K}(\mu_{\text{hard}};\mu_{\text{fact}})\cdot \mathcal{J}_\nu^{NRQCD}(\mu_{\text{fact}}), \tag{1}$$

where the scale μ_{hard} gives the normalization point for the matching of NRQCD with full QCD, while μ_{fact} denotes the normalization point for the calculations in the perturbation theory of NRQCD.

For the heavy quarkonium composed of heavy quarks with the same flavor, the Wilson coefficient $\mathcal{K}$ is known with the two-loop accuracy [13–16]

$$\mathcal{K}(\mu_{\text{hard}};\mu_{\text{fact}}) = 1 - \frac{8}{3}\frac{\alpha_s^{\overline{\text{MS}}}(\mu_{\text{hard}})}{\pi} + \left(\frac{\alpha_s^{\overline{\text{MS}}}(\mu_{\text{hard}})}{\pi}\right)^2 c_2(\mu_{\text{hard}};\mu_{\text{fact}}), \tag{2}$$

and c_2 is explicitly given in [14, 15]. The anomalous dimension of factor $\mathcal{K}(\mu_{\text{hard}};\mu_{\text{fact}})$ in NRQCD is defined by

$$\frac{d\ln\mathcal{K}(\mu_{\text{hard}};\mu)}{d\ln\mu} = \sum_{k=1}^{\infty}\gamma_{[k]}\left(\frac{\alpha_s^{\overline{\text{MS}}}(\mu)}{4\pi}\right)^k, \tag{3}$$

whereas the two-loop calculations[1] give

$$\gamma_{[1]} = 0, \tag{4}$$

$$\gamma_{[2]} = -16\pi^2 C_F\left(\frac{1}{3}C_F + \frac{1}{2}C_A\right). \tag{5}$$

The initial condition for the evolution of factor $\mathcal{K}(\mu_{\text{hard}};\mu_{\text{fact}})$ is given by the matching of NRQCD current with full QCD at $\mu = \mu_{\text{hard}}$ [14, 15].

The leptonic constant of heavy quarkonium is defined in the following way:

$$\langle 0|J_\nu^{QCD}|\bar{Q}Q,\lambda\rangle = \epsilon_\nu^\lambda f_{\bar{Q}Q} M_{\bar{Q}Q}, \tag{6}$$

[1] We use ordinary notations for the invariants of $SU(N_c)$ representations: $C_F = \frac{N_c^2-1}{2N_c}$, $C_A = N_c$, $T_F = \frac{1}{2}$, n_f is a number of “active” light quark flavors.

where λ denotes the vector state polarization ϵ_ν. In full QCD the electromagnetic current of quarks is conserved, while in NRQCD the current $\mathcal{J}_\nu^{NRQCD}$ has the nonzero anomalous dimension, so that in accordance with (1)–(5), we find

$$\langle 0|\mathcal{J}_\nu^{NRQCD}(\mu)|\bar{Q}Q,\lambda\rangle = \mathcal{A}(\mu)\ \epsilon_\nu^\lambda f_{\bar{Q}Q}^{NRQCD} M_{\bar{Q}Q}, \tag{7}$$

where, in terms of nonrelativistic quarks, the leptonic constant for the heavy quarkonium is given by the well-known relation with the wave function at the origin

$$f_{\bar{Q}Q}^{NRQCD} = \sqrt{\frac{12}{M}}\ |\Psi_{\bar{Q}Q}(0)|, \tag{8}$$

and the value of wave function in the leading order is determined by the solution of Schrödinger equation with the static potential, so that we isolate the scale dependence of NRQCD current in the factor $\mathcal{A}(\mu)$, while the leptonic constant $f_{\bar{Q}Q}^{NRQCD}$ is evaluated at a fixed normalization point $\mu = \mu_0$, which will be attributed below. It is evident that

$$f_{\bar{Q}Q} = f_{\bar{Q}Q}^{NRQCD}\mathcal{A}(\mu_{\rm fact})\cdot\mathcal{K}(\mu_{\rm hard};\mu_{\rm fact}), \tag{9}$$

and the anomalous dimension of $\mathcal{A}(\mu_{\rm fact})$ should compensate the anomalous dimension of factor $\mathcal{K}(\mu_{\rm hard};\mu_{\rm fact})$, so that in two loops we have got

$$\frac{d\ln\mathcal{A}(\mu)}{d\ln\mu} = -\gamma_{[2]}\left(\frac{\alpha_s^{\overline{\rm MS}}(\mu)}{4\pi}\right)^2. \tag{10}$$

The physical meaning of $\mathcal{A}(\mu)$ is clearly determined by the relations of (7) and (9): this factor gives the normalization of matrix element for the current of nonrelativistic quarks expressed in terms of wave function for the two-particle quark state (in the leading order of inverse heavy quark mass in NRQCD). Certainly, in this approach the current of nonrelativistic quarks is factorized from the quark-gluon sea, which is a necessary attribute of hadronic state, so that, in general, this physical state can be only approximately represented as the two-quark bound state. In the consideration of leptonic constants in the framework of NRQCD, this approximation requires to introduce the normalization factor $\mathcal{A}(\mu)$ depending on the scale.

The renormalization group equation of (10) is simply integrated out, so that

$$\mathcal{A}(\mu) = \mathcal{A}(\mu_0)\ \left[\frac{\beta_0+\beta_1\dfrac{\alpha_s^{\overline{\rm MS}}(\mu)}{4\pi}}{\beta_0+\beta_1\dfrac{\alpha_s^{\overline{\rm MS}}(\mu_0)}{4\pi}}\right]^{\dfrac{\gamma_{[2]}}{2\beta_1}}, \tag{11}$$

where $\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f$, and $\beta_1 = \frac{34}{3}C_A^2 - 4C_F T_F n_f - \frac{20}{3}C_A T_F n_f$. A constant of integration could be defined so that at a scale μ_0 we would get $\mathcal{A}(\mu_0) = 1$. Thus, in the framework of NRQCD we have got the parametric dependence of leptonic constant estimates on the scale μ_0, which has the following simple interpretation: the normalization of matrix element for the current of nonrelativistic quarks at μ_0 is completely given by the wave function of two-quark bound state. At other $\mu \neq \mu_0$ we have to introduce the factor $\mathcal{A}(\mu) \neq 1$, so that the approximation of hadronic state by the two-quark one becomes inexact.

The relations above are general for the NRQCD description of leptonic constant for the heavy quarkonium in both the approach of sum rules and the calculations with the static potential, so that the only difference is the use of (8) in the potential approach. In the sum rules this relation is substituted by the equations for the constants $f_{\bar{Q}Q}^{NRQCD}$, which follow from the quark-hadron duality in the corresponding scheme of calculations. In this respect, we have to stress that the analysis of NRQCD sum rules for the bottomonium in the two-loop approximation in [5, 7, 16] took into account the factor of $\mathcal{A}(\mu)$ in a different way. So, in the calculation of two-point correlator for the currents of nonrelativistic quarks the regularization is necessary in the two-loop order. Of course, the anomalous dimension of two-point current correlator compensates the anomalous dimension of corresponding Wilson coefficient. In the sum rules with the one-loop gluon correction [4] this factor can be omitted, since its one-loop anomalous dimension is equal to zero, and, hence, the factor can be supposed equal to unit. As we have shown taking into account the two-loop corrections, the calculations without the normalization factor $\mathcal{A}(\mu)$ for the matrix element of NRQCD current would be, in general, not correct.

Some comments are to the point. It is worth to note that we deal with the operator product expansion (OPE), wherein the electromagnetic current of heavy quarks is expressed in terms of leading $1/m_Q$-order current of nonrelativistic spinors near the threshold. Therefore, in this OPE the corresponding Wilson coefficient, i.e. the matching factor $\mathcal{K}$, appears. The first problem is a calculation of this factor. The second is an evaluation of matrix element for the NRQCD current of nonrelativistic quarks. These two problems should be clearly distinguished in the estimates. The calculation of Wislon coefficient $\mathcal{K}$ can be done perturbatively in two ways. Originally, it was done in the technique of threshold expansion of loop integrals for Feynman diagrams in full QCD for heavy quarks [13–16]. We use this result throughout this paper. Further, the second way of $\mathcal{K}$ calculation is the following: one should perturbatively expand the QCD Lagrangian for the interacting heavy quark and heavy anti-quark separated by a distance r in terms of nonrelativistic spinors and represent the perturbative potential contributions different from the interactions with soft and ultrasoft fields in the hadron. These soft terms are not essential in the perturbative calculations of matching factor $\mathcal{K}$ at high virtualities, and under such conditions the system of heavy quark and heavy anti-quark can be considered as the coulomb system in the leading approximation, while the potential corrections such as the spin-dependent and relativistic terms in the form of $1/m_Q$ contributions converted to the α_s corrections to the energy, since in the coulomb systems the distance behaves as $r \sim 1/\alpha_s m_Q \sim 1/|\mathbf{p}|$. Anyway, the main point is that the factor $\mathcal{K}$ can be calculated in the *perturbative theory*. It is important that the fact of cancellation of anomalous dimension for the factor $\mathcal{K}$ by the anomalous dimension for the NRQCD current follows from the zero anomalous dimension of electromagnetic current in full QCD with no reference to the calculation procedure, of course. Thus, the cancellation has no connection to the problem of evaluation for the hadronic matrix element of NRQCD current. Indeed, the effective expansion of QCD Lagrangian for the heavy quarks in the form of potential nonrelativistic QCD (pNRQCD) [17] or velocity-counting nonrelativistic QCD (vNRQCD, discussed below) [18–20] cannot be applied to such the calculation of matrix element because the leading coulomb approximation does not dominate in comparison with corrections for the real quarkonia, even for the $\bar{b}b(1S)$. So, in pNRQCD and vNRQCD the potentials are treated as Wilson coefficients in front of four quark operators, while the soft and ultrasoft terms are separated. This separation is valid only if the time for the formation of coulomb wave function is many times less than the time of interaction with the soft fields (a quark-gluon

sea and a string). Then,

$$T_{coul} \ll T_{soft}.$$

These times can be easily evaluated in terms of distance between the quarks, their velocity and soft scale Λ_{QCD}, so that

$$\left.\begin{array}{r} T_{coul} \sim \dfrac{r}{v} \sim \dfrac{1}{m_Q v^2} \\ T_{soft} \sim \dfrac{1}{\Lambda_{QCD}} \end{array}\right\} \Longrightarrow \quad m_Q v^2 \gg \Lambda_{QCD}.$$

Therefore, in order to allow one to correctly use the pNRQCD or vNRQCD to the description of matrix elements, the kinetic energy of heavy quark motion should be many times greater than the scale of confinement.

In practice, estimates for the most deep 'coulomb' system $b\bar{b}(1S)$ give

$$m_Q v^2 \sim 500 \text{ MeV}, \quad \text{at } \Lambda_{QCD} \sim 300 \text{ MeV}.$$

Thus, in order to get the potential model applicable to the real quarkonia, one should include *soft terms* in the *static* energy, i.e. the *static potential*. This static potential has an infrared stability and it is independent of any separation of soft and non-soft terms in the static energy. The static potential defined in terms of Wilson loop possesses these properties and it does not coincide with the potential (Wilson coefficient) in pNRQCD or vNRQCD, since the infrared contributions are added, but separated. Therefore, we deal with the model, wherein the total energy of all dynamical fields in the presence of static sources is taken into account. By the way, the relativistic and spin-dependent corrections in this case determine the scale of corrections about 50 MeV and their contribution gives the uncertainty of estimation, which is many times less than the uncertainty due to the scale variation in the estimates for the leptonic constants with the static potential.

We stress that there is an ordinary situation in the leptonic constant calculation, since the perturbative technique used to evaluate the Wislon coefficient cannot be explored in order to get a correct estimate for the hadronic matrix element for the operator in front of the Wislon coefficient.

Recently, the effective theory for the heavy nonrelativistic quark and antiquark pair in QCD was developed on the basis of relative velocity expansion [18–20]. This approach uses the so-called velocity renormalization group, so that we refer to such the theory as vNRQCD. The physical argumentation of vNRQCD is the following: In the heavy quark-antiquark system one expects a low influence of light quark-gluon sea formed by nonperturbative emission of quarks and gluons with virtualities about Λ_{QCD}, so that the heavy quark motion is close to the coulomb approximation up to higher order corrections in α_s. Moreover, we can suppose the coulomb relations between various scales, which possess the following counting rules: the relative velocity $v \sim \alpha_s$, the momentum transfer (or the inverse distance between the quarks), i.e. the soft scale $\mu_S \sim |\boldsymbol{k}| \sim m_Q \cdot v$, the momentum of heavy quark $|\boldsymbol{p}| \sim \mu_S$, while the kinetic and potential energies, i.e. the ultrasoft scale, $\mu_U \sim E \sim V \sim m_Q \cdot v^2$, so that in the perturbative part of effective Lagrangian the distance, momentum and energy are not free independent quantities, since they are related by the single "running" parameter v. This approach is certainly different from pNRQCD, wherein the static bilinear operators of nonrelativistic spinors are fixed at a short distance, while the kinetic energy as well as the quark momenta are put to zero.

Thus, the starting point of vNRQCD is the correlation of soft and ultrasoft scales, $\mu_U \approx v \cdot \mu_S$, in contrast to pNRQCD. Therefore, in vNRQCD the matching with the full QCD at the scale of

heavy quark mass $\mu_{\rm hard} = m_Q$ and the renormalization group evolution with v involving large $\ln v$ result in a combined summation[2] of relevant momentum and energy contributions from $\mu = m_Q$ up to $\mu = m_Q \cdot v^2$, while pNRQCD deals with the two-step matching and evolution: NRQCD matched with full QCD at $\mu_{\rm hard} \sim m_Q$ and evolved to $\mu_S \sim m_Q \cdot v$, and pNRQCD matched with NRQCD at μ_S and evolved to $\mu \sim m_Q \cdot v^2$. A peculiar feature of vNRQCD summing up both logs of μ_S and μ_U is that the limits of $m_Q \to \infty$ or $v \to 0$ do not reproduce the static limit (see explanations provided by examples in [20]).

Next point is the anomalous dimension of nonrelativistic current, that was calculated in [18,19] in the technique of vNRQCD. One found that expressions (3)-(5) are reproduced in vNRQCD and strictly valid at $\mu = m_Q$, while at lower virtualities with $v = \mu_{\rm fact}/m_Q < 1$ some corrections in $\ln v$ appear. These corrections can be summed up in vNRQCD, so that substitution of $\alpha_s^2(\mu)$ for $\alpha_s^2(m_Q)$ in the two-loop anomalous dimension could be quite naive and not strictly justified, by opinion in [19]. The analogous two-loop anomalous dimension γ of $\mathcal{K}$ in vNRQCD as well as the complete expression for $\mathcal{K}^{\rm vNRQCD}(v)$ integrated out in the velocity renormalization group are available in [19]. So,

$$\mathcal{K}^{\rm vNRQCD}(v) = \mathcal{K}^{\rm vNRQCD}(1)\, \exp\left[\sum_{i=1}^{7} a_i^v \pi \alpha_s(m_Q) k_i\right], \tag{12}$$

where a_i^v are expressed in terms of C_F, C_A and β_0, and the functions k_i have the form

$$\begin{aligned}
k_1 &= \frac{1}{z} - 1, \qquad\qquad k_2 = 1 - z, \quad k_3 = \ln(z),\\
k_4 &= 1 - z^{1-13C_A/(6\beta_0)}, \quad k_5 = 1 - z^{1-2C_A/\beta_0},\\
k_6 &= \frac{\pi^2}{12} - \frac{1}{2}\ln^2(2) - \ln(w)\ln\left(\frac{2w}{2w-1}\right) - {\rm Li}_2\left(\frac{1}{2w}\right),\\
k_7 &= \frac{w}{2w-1}\ln(w) - \frac{1}{2}\ln(2w-1),
\end{aligned}$$

while $z = \alpha_s(\mu)/\alpha_s(m_Q)$, $w = \alpha_s(\mu^2/m_Q)/\alpha_s(\mu)$, and the one-loop matching condition is given by the same form as in (2), i.e.

$$\mathcal{K}^{\rm vNRQCD}(1) = 1 - 2C_F\,\frac{\alpha_s(m_Q)}{\pi}. \tag{13}$$

However, in general the scale prescription in the argument of α_s standing in the first nonzero term, which presents the accuracy under study, is rather conventional, since the scale change results in the next-order contribution in α_s, that is not under control. Indeed, the substitution

$$\alpha_s(\mu) \approx \alpha_s(m_Q)\left(1 - \frac{\beta_0}{2\pi}\,\alpha_s(m_Q)\,\ln\frac{\mu}{m_Q}\right) = \alpha_s(m_Q)\left(1 - \frac{\beta_0}{2\pi}\,\alpha_s(m_Q)\,\ln v\right),$$

influences the three-loop anomalous dimension γ for $\mathcal{K}$, only. The argumentation of vNRQCD is that the corrections summing up $[\ln v]^n$ are dominant (in all orders in α_s). So, the above note on the scale substitution manifests itself in three-loop running of coulomb-like potentials in vNRQCD.

In the present paper we follow the static ideology with the threshold expression for $\mathcal{K}(\mu)$ (and hence, $\mathcal{A}(\mu)$), since we adopt the Schrödinger equation with the model of static potential for the

[2]Note that terms $[\ln v]^n \sim [\ln \alpha_s]^n$ reproduce so-called manifestly nonanalytic contributions in α_s-expansion.

$\bar{Q}Q$ system in contrast to the coulomb approximation improved by the velocity renormalization group involving running coulomb and relevant potentials. Moreover, the threshold expression for $\mathcal{K}(\mu)$ is preferable by technical reasons, since, first, it is valid at the two-loop order (as we have mentioned above), second, $\mathcal{K}(\mu)$ has an explicit dependence on the matching scale $\mu_{\rm hard}$, that allows us to study the point stable under the variation of $\mu_{\rm hard}$ (see below), while in vNRQCD the matching scale is fixed and prescribed to $\mu_{\rm hard} = m_Q$, and the corresponding analysis on the stability is not possible. Third, the factor $\mathcal{K}(\mu)$ contains the two-loop final term independent of $\ln\mu$, while $\mathcal{K}^{\rm vNRQCD}(v)$ does not (except the one-loop term), since in the large $\ln v$ approximation the two-loop final term is irrelevant. This two-loop final term in $\mathcal{K}(\mu)$ makes the cancellation of μ-dependence in the product of $\mathcal{K}(\mu)\cdot\mathcal{A}(\mu)$ valid in a restricted region of μ, that is conceptually true, while in vNRQCD the large log approximation results in the complete cancellation of v-dependence (remember, $v = \mu_{\rm fact}/m_Q$) in the product of $\mathcal{K}^{\rm vNRQCD}(\mu)\cdot\mathcal{A}^{\rm vNRQCD}(\mu)$. Nevertheless, since we assume the existence of μ_0, at which $\mathcal{A}(\mu_0) = 1$, then we can estimate the above product by $\mathcal{K}^{\rm vNRQCD}(v_0)$. The problem on the choice of μ_0 in vNRQCD is more artificial than in NRQCD, since the analysis of stability on $\mu_{\rm hard}$ results in a preferable prescription for μ_0 in NRQCD, while in vNRQCD we have no analogous criterion (see Section 4 with numerical results).

3 Potential approach

The potential of static heavy quarks illuminates the most important features of QCD dynamics: the asymptotic freedom and confinement. In the leading order of perturbative QCD at short distances and with a linear confining term in the infrared region, the potential of static heavy quarks was considered in the Cornell model [21], incorporating the simple superposition of both asymptotic limits (the effective coulomb and string-like interactions). The observed heavy quarkonia posed in the intermediate distances, where both terms are important for the determination of mass spectra (see Fig. 1). So, the phenomenological approximations of potential (logarithmic one [22] and power law [23]), taking into account the regularities of such the spectra, were quite successful [24].

The quantities more sensitive to the global properties of potential are the wave functions at the origin as related to the leptonic constants and production rates. So, the potentials consistent with the asymptotic freedom to one and two loops as well as the linear confinement were proposed by Richardson [25], Buchmüller and Tye [12], respectively.

In QCD the static potential is defined in a manifestly gauge invariant way by means of the vacuum expectation value of a Wilson loop [26],

$$\begin{aligned} V(r) &= -\lim_{T\to\infty}\frac{1}{iT}\ln\langle\mathcal{W}_\Gamma\rangle\,, \\ \mathcal{W}_\Gamma &= \widetilde{\rm tr}\,\mathcal{P}\exp\left(ig\oint_\Gamma dx_\mu A^\mu\right)\,. \end{aligned} \tag{14}$$

Here, Γ is taken as a rectangular loop with time extension T and spatial extension r. The gauge fields A_μ are path-ordered along the loop, while the color trace is normalized according to $\widetilde{\rm tr}(..) = {\rm tr}(..)/{\rm tr}\mathbb{1}$. This definition corresponds to the calculation of effective action for the case of two external sources fixed at a distance r during an infinitely long time period T, so that the time-ordering coincides with the path-ordering. Moreover, the contribution into the effective action by the path parts, where the charges have been separated to the finite distance during a finite time, can be neglected in comparison with the infinitely growing term of $V(r)\cdot T$. Let us emphasize that

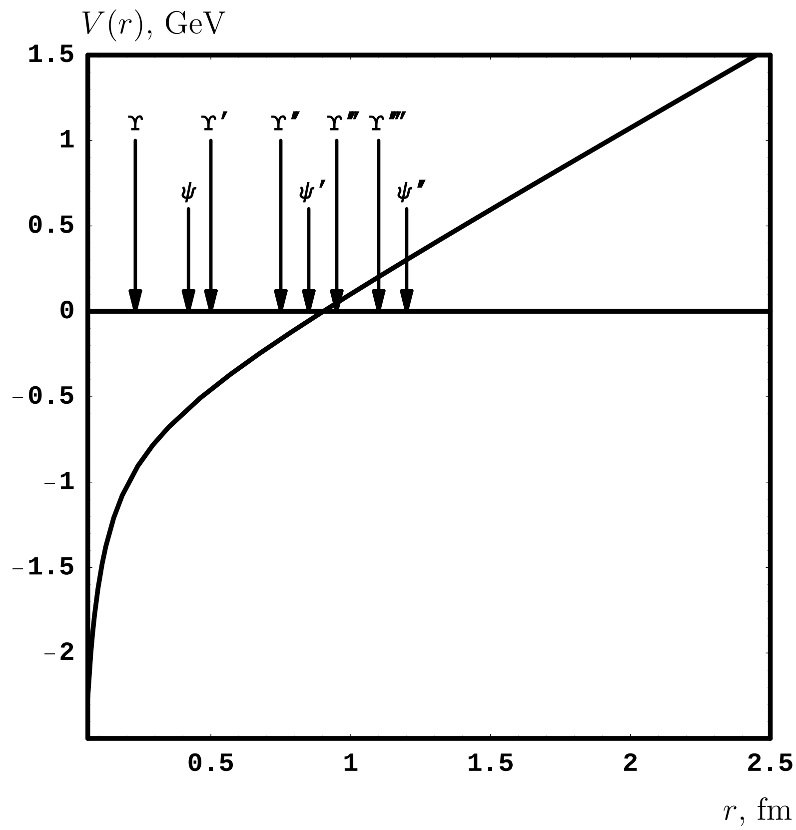

Figure 1: The Cornell model of static potential and sizes of observed heavy quarkonia with charmed quarks (the family ψ) and bottom quarks (the family Υ).

the defined static potential is, by construction, the renormalization invariant quantity, since the action, by definition, doe not depend on the normalization point.

Generally, one introduces the V scheme of QCD coupling constant by the definition of QCD potential of static quarks in momentum space as follows:

$$V(\mathbf{q}^2) = -C_F \frac{4\pi\alpha_{\mathrm{V}}(\mathbf{q}^2)}{\mathbf{q}^2}, \tag{15}$$

so that for the such-way introduced value of α_{V} one can derive some results at large virtualities in the perturbative QCD as well as at low transfer momenta in the approximation of linear term in the potential confining the quarks.

In this section, first, we discuss two regimes for the QCD forces between the static heavy quarks: the asymptotic freedom and confinement. Then we follow the method by Buchmüller and Tye and formulate how these regimes can be combined in a unified β function for α_{V} obeyed both limits of small and large QCD couplings.

In the perturbative QCD up to the two-loop accuracy, the quantity α_{V} can be matched with $\alpha_{\overline{\mathrm{MS}}}$ by the relation

$$\alpha_{\mathrm{V}}(\mathbf{q}^2) = \alpha_{\overline{\mathrm{MS}}}(\mu^2) \sum_{n=0}^{2} \tilde{a}_n(\mu^2/\mathbf{q}^2) \left(\frac{\alpha_{\overline{\mathrm{MS}}}(\mu^2)}{4\pi}\right)^n = \alpha_{\overline{\mathrm{MS}}}(\mathbf{q}^2) \sum_{n=0}^{2} a_n \left(\frac{\alpha_{\overline{\mathrm{MS}}}(\mathbf{q}^2)}{4\pi}\right)^n. \tag{16}$$

At present, in (16) the coefficients of tree approximation a_0, one-loop contribution a_1 and new two-loop results a_2 (see [10, 11]) are known. Note that expansion (16) cannot be straightforwardly extended to higher orders of perturbative QCD because of infrared problems, that result in non-analytic terms in the three-loop perturbative potential as was first discussed by Appelquist, Dine and Muzinich [26].

After the introduction of $\mathfrak{a} = \frac{\alpha}{4\pi}$, the β function is actually defined by

$$\frac{d\mathfrak{a}(\mu^2)}{d\ln\mu^2} = \beta(\mathfrak{a}) = -\sum_{n=0}^{\infty} \beta_n \cdot \mathfrak{a}^{n+2}(\mu^2), \tag{17}$$

so that $\beta^{\rm V}_{0,1} = \beta^{\overline{\rm MS}}_{0,1}$ and $\beta^{\rm V}_2 = \beta^{\overline{\rm MS}}_2 - a_1\beta^{\overline{\rm MS}}_1 + (a_2 - a_1^2)\beta^{\overline{\rm MS}}_0$.

The Fourier transform results in the position-space potential [10], so that the perturbative potential, by construction, is independent of normalization point, i.e. it is the renormalization group invariant. However, in the problem under consideration the truncation of perturbative expansion, wherein the coefficients do not decrease[3], leads to a strong custodial dependence on the normalization point. So, putting the normalization point μ in the region of charmed quark mass, we find that the two-loop potential with the three-loop running coupling constant $\alpha_s^{\overline{\rm MS}}$ has an unremovable additive shift depending on μ. This shift has variation in wide limits. This fact illuminates the presence of infrared singularity in the coupling constant of QCD, so that the μ-dependent shift in the potential energy has the form of a pole posed at Λ_{QCD} [9].

Thus, in order to avoid the ambiguity of static potential in QCD we have to deal with infrared stable quantities. The motivation by Buchmüller and Tye was to write down the β function of $\alpha_{\rm V}$ consistent with two known asymptotic regimes at short and long distances. They proposed the function, which results in the effective charge determined by two parameters, only: the perturbative parameter is the scale in the running of coupling constant at large virtualities and the nonperturbative parameter is the string tension. The necessary inputs are the coefficients of β function. The parameters of potential by Buchmüller and Tye were fixed by fitting the mass spectra of charmonium and bottomonium [3]. Particularly, in such the phenomenological approach the scale $\Lambda^{n_f=4}_{\overline{\rm MS}} \approx 510$ MeV was determined. It determines the asymptotic behaviour of coupling constant at large virtualities in QCD. This value is in a deep contradiction with the current data on the QCD coupling constant $\alpha_s^{\overline{\rm MS}}$ [3]. In addition, one can easily find that the three-loop coefficient $\beta^{\rm V}_2$ for the β function suggested by Buchmüller and Tye is not correct even by its sign and absolute value in comparison with the exact coefficient recently calculated in [10, 11].

Thus, the modification of Buchmüller–Tye (BT) potential of static quarks as dictated by the current status of perturbative calculations is of great interest.

The nonperturbative behaviour of QCD forces between the static heavy quarks at long distances r is usually represented by the linear potential (see discussion in ref. [27])

$$V^{\rm conf}(r) = k \cdot r, \tag{18}$$

which corresponds to the square-law limit for the Wilson loop. The form of (18) corresponds to the limit, when at low virtualities $\mathbf{q}^2 \to 0$ the coupling $\alpha_{\rm V}$ tends to

$$\alpha_{\rm V}(\mathbf{q^2}) \to \frac{K}{\mathbf{q^2}},$$

[3]Moreover, according to the investigations of renormalon, the coefficients in the series of perturbation theory for the potential increase in the factorial power, so that the series has a meaning of asymptotic one.

so that

$$\frac{d\alpha_{\rm V}(\mathbf{q^2})}{d\ln\mathbf{q^2}} \to -\alpha_{\rm V}(\mathbf{q^2}), \tag{19}$$

which gives the confinement asymptotics for the $\beta_{\rm V}$ function.

Buchmüller and Tye proposed the procedure for the reconstruction of β function in the whole region of charge variation by the known limits of asymptotic freedom to a given order in α_s and confinement regime. Generalizing their method, the $\beta_{\rm PT}$ function found in the framework of asymptotic perturbative theory (PT) to three loops, is transformed to the β function of effective charge as follows:

$$\begin{aligned}
\frac{1}{\beta_{\rm PT}(\mathfrak{a})} &= -\frac{1}{\beta_0\mathfrak{a}^2} + \frac{\beta_1 + \left(\beta_2^{\rm V} - \frac{\beta_1^2}{\beta_0}\right)\mathfrak{a}}{\beta_0^2\mathfrak{a}} \Longrightarrow \\
\frac{1}{\beta(\mathfrak{a})} &= -\frac{1}{\beta_0\mathfrak{a}^2\left(1-\exp\left[-\frac{1}{\beta_0\mathfrak{a}}\right]\right)} + \frac{\beta_1 + \left(\beta_2^{\rm V} - \frac{\beta_1^2}{\beta_0}\right)\mathfrak{a}}{\beta_0^2\mathfrak{a}}\exp\left[-\frac{l^2\mathfrak{a}^2}{2}\right],
\end{aligned} \tag{20}$$

where the exponential factor in the second term contributes to the next-to-next-to-leading order at $\mathfrak{a}\to 0$. This function has the essential peculiarity at $\mathfrak{a}\to 0$, so that the expansion is the asymptotic series in $\mathfrak{a}$. At $\mathfrak{a}\to\infty$ the β function tends to the confinement limit represented in (19).

The construction of (20) is based on the idea to remove the pole from the coupling constant at a finite energy, but in contrast to the "analytic" approach developed in [28], the "smothing" of peculiarity occurs in the logarithmic derivative of charge related with the β function, but in the expression for the charge itself. Indeed, in the one-loop perturbation theory we have got

$$\frac{d\ln\mathfrak{a}}{d\ln\mu^2} = -\beta_0\mathfrak{a} = -\frac{1}{\ln\frac{\mu^2}{\Lambda^2}},$$

because of

$$\mathfrak{a} = \frac{1}{\beta_0\ln\frac{\mu^2}{\Lambda^2}},$$

and the pole can be cancelled in the logarithmic derivative itself, so that[4]

$$\frac{d\ln\mathfrak{a}}{d\ln\mu^2} \Longrightarrow -\beta_0\mathfrak{a}\left(1-\frac{\Lambda^2}{\mu^2}\right) \approx -\beta_0\mathfrak{a}\left(1-\exp\left[-\frac{1}{\beta_0\mathfrak{a}}\right]\right).$$

As we see in the perturbative limit $\mathfrak{a}\to 0$ the deviation in the β function is exponentially small. The generalization of this idea to two loops was done by Buchmüller and Tye.

Remember that the one and two-loop static potentials matched with the linear term of confinement lead to the contradiction with the value of QCD coupling constant extracted at the scale of Z boson mass if we fit the mass spectra of heavy quarkonia in such potentials. The static potential in the three-loop approximation results in the consistent value of QCD coupling constant at large virtualities [9].

[4]The presentation of power correction in the form of charge exponential was similarly used in [29].

In the perturbative limit the usual solution for the running coupling constant

$$\mathfrak{a}(\mu^2) = \frac{1}{\beta_0 \ln \frac{\mu^2}{\Lambda^2}} \left[1 - \frac{\beta_1}{\beta_0^2} \frac{1}{\ln \frac{\mu^2}{\Lambda^2}} \ln\ln \frac{\mu^2}{\Lambda^2} + \frac{\beta_1^2}{\beta_0^4} \frac{1}{\ln^2 \frac{\mu^2}{\Lambda^2}} \left(\ln^2 \ln \frac{\mu^2}{\Lambda^2} - \ln\ln \frac{\mu^2}{\Lambda^2} - 1 + \frac{\beta_2^{\rm V}\beta_0}{\beta_1^2}\right)\right], \tag{21}$$

is valid. Using the asymptotic limit of (21), one can get the equation

$$\ln \frac{\mu^2}{\Lambda^2} = \frac{1}{\beta_0 \mathfrak{a}(\mu^2)} + \frac{\beta_1}{\beta_0^2} \ln \beta_0 \mathfrak{a}(\mu^2) + \int_0^{\mathfrak{a}(\mu^2)} dx \left[\frac{1}{\beta_0 x^2} - \frac{\beta_1}{\beta_0^2 x} + \frac{1}{\beta(x)}\right], \tag{22}$$

which can be easily integrated out, so that we get an implicit solution for the charge depending on the scale. The implicit equation can be inverted by the iteration procedure, so that well approximated solution has the form

$$\mathfrak{a}(\mu^2) = \frac{1}{\beta_0 \ln \left(1 + \eta(\mu^2) \frac{\mu^2}{\Lambda^2}\right)}, \tag{23}$$

where $\eta(\mu^2)$ is expressed through the coefficients of perturbative β function and parameter l in (20), which is related to the slope of Regge trajectories α'_P and the integration constant, the scale Λ [9].

The slope of Regge trajectories, determining the linear part of potential, is supposed equal to $\alpha'_P = 1.04$ GeV^{-2}, so that in (18) we put the parameter $k = \frac{1}{2\pi\alpha'_P}$. We use also the measured value of QCD coupling constant [3] and pose

$$\alpha_s^{\overline{\rm MS}}(m_Z^2) = 0.123,$$

as the basic input of the potential. The transformation into the configuration space was done numerically in [9], so that the potential is presented in the form of file in the notebook format of MATHEMATICA system.

The analysis of quark masses and mass spectra of heavy quarkonia results in the following values ascribed to the potential approach [9]:

$$m_c^{\rm V} = 1.468 \text{ GeV}, \quad m_b^{\rm V} = 4.873 \text{ GeV}. \tag{24}$$

Thus, the spectroscopic characteristics of systems composed of nonrelativistic heavy quarks are determined in the approach with the static potential described above.

4 Numerical estimates

In the potential approach the values of wave functions for the $1S$ states in the systems of $\bar{b}b$ and $\bar{c}c$ are equal to

$$\sqrt{4\pi}\Psi_{\bar{b}b}(0) = 2.513 \text{ GeV}^{3/2}, \quad \sqrt{4\pi}\Psi_{\bar{c}c}(0) = 0.895 \text{ GeV}^{3/2}, \tag{25}$$

which are the renormalization group invariants by construction of Schrödinger equation with the static potential. Remember that this potential includes the contribution of linear part providing

the confinement of quarks. The same term makes the infrared stability of effective charge and all of the potential in NRQCD.

The analysis of leptonic constants of Υ and ψ with account of Wilson coefficient $\mathcal{K}$, derived in the perturbative NRQCD, was done in [9]. For Υ we found that the variation of hard scale in wide limits $\mu_{\rm hard} = (1-2)m_b$ led to the presence of a stable point $\mu_{\rm fact}$, in which the result was weakly sensitive to the variation of hard scale. The stability was observed at $\mu_{\rm fact} \approx 2.2 - 2.7$ GeV, i.e. at the inter-quark distances characteristic for the size of $1S$ level in the system of $\bar{b}b$.

For the leptonic constant of charmonium J/ψ the stability point under the variation of $\mu_{\rm hard}$ was located at the reasonable values of soft scale of factorization $\mu_{\rm fact} \approx 1.05 - 1.35$ GeV. The position of stable point was significantly dependent of changes in $\mu_{\rm hard}$ close to the mass of charmed quark.

However, in that consideration the stability of results under the variation of scale $\mu_{\rm fact}$ for the perturbative calculations concerning the Wislon coefficient in NRQCD was not observed.

In this paper we introduce the normalization factor $\mathcal{A}$ into the analysis for the current of nonrelativistic quarks. As we see in Fig. 2, in the system of $\bar{b}b$ the choice of initial point of normalization $\mu_0 = 2.3$ GeV in the mentioned region of stability for the leptonic constant under the variation of matching scale, so that $\mathcal{A}(2.3\ \text{GeV}) = 1$, we get the independence of result for the lepton constant of Υ in the interval $\mu_{\rm fact} = 1.8 - 3.8$ GeV.

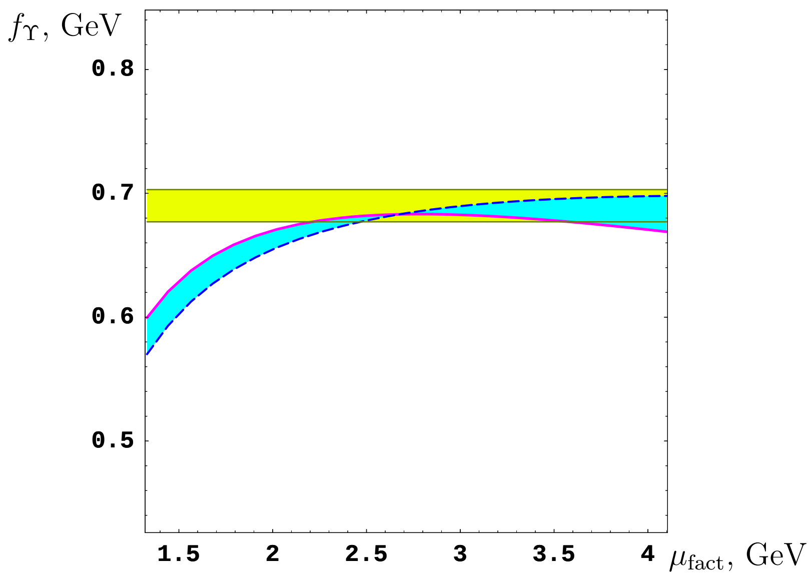

Figure 2: The leptonic constant of ground vector state in the system of bottomonium is presented versus the soft scale of normalization. The dashed curve corresponds to $\mu_{\rm hard} = 2\,m_b$, while the solid curve does to $\mu_{\rm hard} = m_b$. The initial condition for the evolution of normalization factor $\mathcal{A}(\mu_{\rm fact})$ in the matrix element of current in the nonrelativistic representation has been posed in the form $\mathcal{A}(2.3\ \text{GeV}) = 1$. The horizontal band is the experimental limits for the constant.

The analysis of stability is performed in Fig. 3. Thus, our estimate of leptonic constant for the ground vector state of bottomonium is given by

$$f_\Upsilon = 685 \pm 30\ \text{MeV},$$

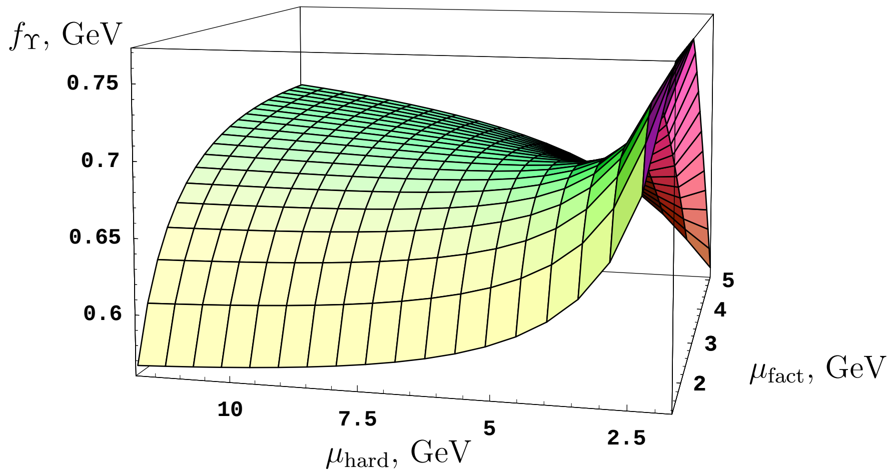

Figure 3: The leptonic constant of ground vector state in the system of bottomonium is presented versus the soft and hard scales, $\mu_{\rm fact}$ and $\mu_{\rm hard}$. The initial condition for the evolution of normalization factor $\mathcal{A}(\mu_{\rm fact})$ in the matrix element of current in the nonrelativistic representation is $\mathcal{A}(2.3\ {\rm GeV}) = 1$.

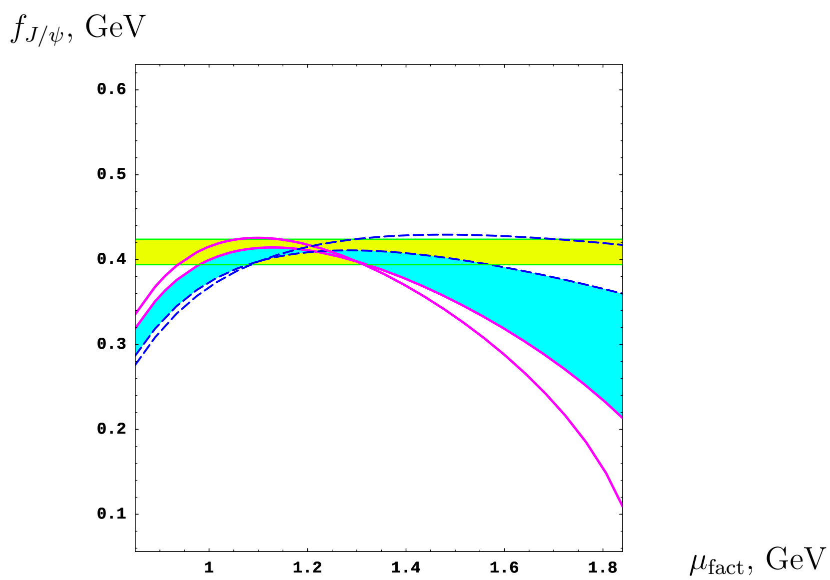

Figure 4: The leptonic constant of ground vector state in the system of charmonium is presented versus the soft scale of normalization. The shaded region restricted by curves corresponds to the change of hard scale from $\mu_{\rm hard} = 2.0\, m_c$ (the dashed curve) to $\mu_{\rm hard} = 1.3\, m_c$ (the solid curve). The initial condition for the evolution of normalization factor $\mathcal{A}(\mu_{\rm fact})$ in the matrix element of current in the nonrelativistic representation has been posed in the form $\mathcal{A}(1.07\ {\rm GeV}) = 1$. The horizontal band is the experimental limits for the constant. The additional curves are presented for $\mu_{\rm hard} = 2.8\, m_c$ (the dashed line) and $\mu_{\rm hard} = 1.15\, m_c$ (the solid line).

which is in a good agreement with the experimental data.

The calculations of leptonic constant for the charmonium J/ψ are more sensitive to the choice of initial condition in the evolution of factor $\mathcal{A}$, because the stability point under the variation of hard scale for the matching is significantly fluctuates under the change of $\mu_{\rm hard} = (1-2)\, m_c$. In Fig. 4 we show the dependence of $f_{J/\psi}$ on the scale $\mu_{\rm fact}$ at $\mathcal{A}(1.07~\text{GeV}) = 1$. This value of $\mu_0 = 1.07$ GeV agrees with the inverse size of system $\bar{c}c$. We see in the figure that the region of result stability is given by $\mu_{\rm fact} = 1. - 1.6$ GeV.

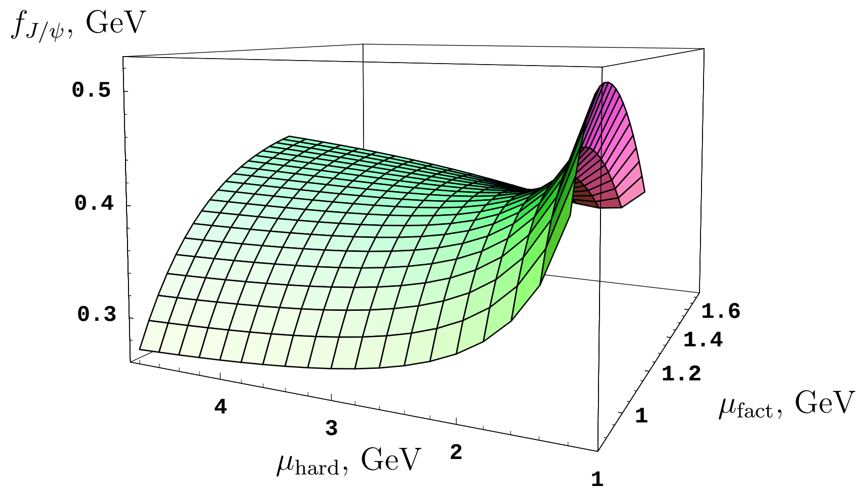

Figure 5: The leptonic constant of ground vector state in the system of charmonium is presented versus the soft and hard scales, $\mu_{\rm fact}$ and $\mu_{\rm hard}$. The initial condition for the evolution of normalization factor $\mathcal{A}(\mu_{\rm fact})$ in the matrix element of current in the nonrelativistic representation is given by $\mathcal{A}(1.07~\text{GeV}) = 1$.

The dependence of estimates on the scales are presented in Fig. 5. Then we get

$$f_{J/\psi} = 400 \pm 45~\text{MeV},$$

which can be compared with the experimental value $f^{\rm exp}_{J/\psi} = 409 \pm 15$ MeV.

Thus, the analysis of stability for the leptonic constants of heavy quarkonia in the framework of potential approach shows that there is the region of stability under the change of both factorization scale in the perturbative NRQCD and point marking the matching of effective theory of nonrelativistic quarks with the full QCD. The basic source of uncertainty in the estimates is the variation of initial normalization point for the evolution factor determining the matrix element for the current of nonrelativistic quarks, so that the values of leptonic constants in the extremum (see Figs. 3 and 5) change in the limits, which are included into the methodic error of estimates as shown above.

In vNRQCD the factor $\mathcal{K}^{\rm vNRQCD}(v)$ demonstrates the stability region (see Fig. 6 for the bottomonium). This stable point is the only reasonable indication for the choice of v_0, at which $\mathcal{A}^{\rm vNRQCD}(v_0) = 1$. Such the prescription results in the estimates of leptonic constants derived in vNRQCD

$$f_\Upsilon\big|_{\rm vNRQCD} = 680 \pm 30~\text{MeV}, \quad f_{J/\psi}\big|_{\rm vNRQCD} = 360 \pm 45~\text{MeV}, \tag{26}$$

where the errors are dominantly due to the variations of α_s at the masses of bottom and charmed quarks: $\alpha_s(m_b) = 0.22 \pm 0.02$, $\alpha_s(m_c) = 0.34 \pm 0.05$ entering the one-loop matching condition. We see that the vNRQCD estimates are in agreement with both the static values and experimental data.

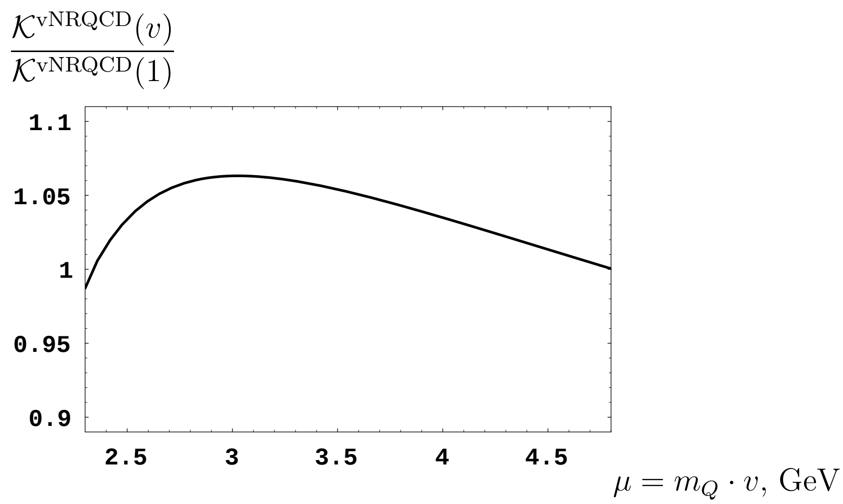

Figure 6: The Wilson coefficient $\mathcal{K}^{\rm vNRQCD}(v)$ for the vector electromagnetic current of nonrelativistic bottom quarks in vNRQCD.

Finally, in the potential approach we can calculate the ratios of leptonic constants for the excited nS-wave levels in $\bar{b}b$ and $\bar{c}c$, which were presented in [9] in comparison with the experimental data. These theoretical calculations are in a good agreement with the measured values.

5 Conclusion

In the present work we have done the general consideration of problem on the calculation of leptonic constants for the heavy quarkonia in NRQCD and introduced the factor for the normalization of matrix element for the current of nonrelativistic quarks. This factor has got the anomalous dimension of current in the effective theory, so that the explicit invariance under the action of renormalization group takes place for the estimated values of leptonic constants. In the framework of potential approach we have estimated the constants for f_Υ and $f_{J/\psi}$, so that we have found reasonable regions of stability under the variation of both the scale for the matching of NRQCD with full QCD and the soft scale ascribed to the operators of nonrelativistic quarks in NRQCD. The obtained results are in a good agreement with the experimental data.

The authors expresses their gratitude to V.A.Rubakov and S.A.Larin for the stimulating remarks and the challenge, the response to which leads to the writing of this work.

This work is in part supported by the Russian Foundation for Basic Research, grants 01-02-99315, 01-02-16585 and 00-15-96645 and the Federal program “State support for the integration of high education and fundamental science”, grant 247 (V.V.K. and O.N.P.).

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